Autumn 2012 **Extremal Combinatorics - Sheet 3** E.P. Long

There are also exercises in the notes; these are intended to assist in your understanding of the material and should all be attempted. The examples sheets are unassessed, but you are welcome to hand in your attempts the week after they are handed out for feedback.

1. Show that the Local LYM inequality follows from the LYM inequality.

2. Conversely, show that in fact the LYM inequality follows from the Local LYM inequality. (*Hint: using Local LYM, work your way down from the top layer, one layer at a time.*)

3. Show that if equality holds in the Local LYM inequality then $\mathcal{A}$ is either empty or the complete layer $\binom{X}{r}$.

4. When does equality hold in the LYM inequality? (*Hint: use the previous two excercises*)

5. Show that if equality holds in Sperner's theorem then the set system is a middle layer, that is $\mathcal{A} = \binom{X}{k}$ for $k = \lfloor n/2 \rfloor$ or $k = \lceil n/2 \rceil$.

6. Suppose that $\mathcal{A} \subset \mathcal{P}[n]$ is a family of sets which does not contain three distinct sets A, B and C with $A \subset B \subset C$. How big can $|\mathcal{A}|$ be?

7. Write out $\binom{[5]}{3}$ ordered by colex. How does this relate to $\binom{[6]}{3}$ ordered by colex?

8. Let $r \in \mathbb{N}$. Show that any positive integer m can be written uniquely in the form
$$m = \binom{m_r}{r} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s}$$
where $m_r > m_{r-1} > \cdots > m_s > 0$ and $r \geq s \geq 1$. (*Hint: think greedily.*)

9. Show that if $\mathcal{C}$ is an initial segment of colex of size $|\mathcal{C}| = \binom{m_r}{r} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s}$ as above then $|\partial\mathcal{C}| = \binom{m_r}{r-1} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s-1}$.

10. Show that an initial segment of colex is left-compressed.

Please mail me if you have any comments or corrections.

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