Assessed Example Sheet 2. MSM3A05/MSM4A05
Due to be handed in at 10am Tuesday 27th November.

QUESTION 1. [8 MARKS]

Consider the integral

$$I(x) = \int_0^{2\pi} e^{ix\sin\theta} d\theta,$$

where $x \in \mathcal{R}$. Using the method of stationary phase show that

$$I \sim 2\sqrt{\frac{2\pi}{x}}\cos(x - \pi/4) \sim \sqrt{\frac{8\pi}{x}}\cos(x - \pi/4),$$

as $x \to \infty$.

QUESTION 2. [8 MARKS]

Consider the problem

$$\epsilon\frac{d^2y}{dx^2} + x^n\frac{dy}{dx} - x^m y = 0, \quad 0 < \epsilon \ll 1, \quad 0 < x < 1, \quad y(0) = \alpha, \quad y(1) = \beta, \ (1)$$

where α, β, n and m are real constants.

Assume that a boundary layer exists at $x = 0$.

(a) Find a one term outer solution.

(b) Re-scale using an inner variable and obtain distinguished limits of equation (1) [Hint: there are three distinguished limits].

(c) Find the conditions under which no distinguished limit exists and therefore there is no boundary layer at $x = 0$.

JU 10/11/12