Assessed Example Sheet 3. MSM3A05/MSM4A05
Due to be handed by Monday 3rd December at 10 am.

QUESTION 1.
[18 MARKS] Consider the boundary value problem

$$\epsilon(2y + y'') + 2xy' - 4x^2 = 0 \quad \text{for} \quad -1 \leq x \leq 2,$$

subject to

$$y(-1) = 2, \quad y(2) = 7,$$

with $\epsilon \ll 1$.

(a) Show that it is not possible to have a boundary layer at either $x = -1$ or $x = 2$. Now assume that an interior layer exists at $x = 0$.
(b) Find the leading order outer solution away from this interior layer.
(c) Determine the re-scaling needed for an interior layer at $x = 0$. Find the leading order inner solution.
(d) Match these two solutions, and hence, find the leading order expression for $y(0)$ as $\epsilon \to 0$.
(e) Sketch the leading order solution.

QUESTION 2. **This question is only for fourth year students!!**
[10 MARKS]

(a) Write a few paragraphs about a physical problem involving boundary layers. You should be descriptive and attempt to include some mathematics.

(b) Considering the equation

$$\epsilon y'' + 2y' + 2y = 0,$$

subject to

$$y(0) = 0, \quad y(1) = 1,$$

with $\epsilon \ll 1$.
Attempt to show why it would not make sense to have any interior layers - that is why could there not be an interior layer at $x = x_0$ where $x_0 \in (0, 1]$.

JU 23/11/12