MSM3A05a/MSM4A05a Problem Sheet 1.

QUESTION 1.

Calculate the order of magnitude of the following functions as $z \to 0$ (i.e., find a value for m such that these functions are $O(z^m)$)

- (a) $\sinh z$
- (b) $\tan z$
- (c) $\sinh\left(\frac{1}{z}\right)$
- (d) e^{-z}
- (e) $\cot z$
- (f) $\ln(1+z)$
- (g) $\frac{1-\cos z}{1+\cos z}$
- (h) $e^{-\cosh\frac{1}{z}}$

QUESTION 2. Show that the following relations are valid.

- (a) $1-\cos^2 z = O(z^2)$ as $z \to 0$
- (b) $1-\cos^2 z = o(z)$ as $z \to 0$
- (c) $\cos z = o(z^{-\frac{1}{3}})$ as $z \to 0$
- (d) $(\ln z)^2 = o(z^{\frac{1}{3}})$ as $z \to \infty$
- (e) $e^{-\frac{1}{z}} = o(z^n)$ for all n as $z \to 0$
- (c) $\sinh z^{-1} = o(1)$ as $z \to \infty$

QUESTION 3. Arrange the following in descending order for small z

$$z^2, z^{\frac{1}{2}}, \ln(\ln z^{-1}), 1, z^{\frac{1}{2}} \ln z^{-1}, z \ln z^{-1}, e^{-\frac{1}{z}}, \ln z^{-1}, z^{\frac{3}{2}}, z, z^2 \ln z^{-1}$$

QUESTION 4 Show that the following are asymptotic sequences:

1. $\left(\sin\frac{1}{z}\right)^n$, $n = 0, 1, \ldots$, $z \to \infty$
2. $\ln(1+z^n)$, $n = 0, 1, \ldots$, $z \to 0$.
3. $(z-z_0)^n$ for $n = 0, 1, 2, 3...$ as $z \to z_0$

QUESTION 5 Arrange the following terms in descending order for small ϵ:

$$\epsilon^{\frac{1}{2}} \ln \epsilon, \quad \epsilon, \quad e^{e^{\frac{1}{\epsilon}}}, \quad e^{-\frac{1}{\epsilon}}, \quad -\ln \epsilon, \quad 5^{-\frac{1}{\epsilon}}, \quad \epsilon^{-\frac{3}{2}}, \quad \epsilon^{-0.0001}.$$

QUESTION 6 Find an asymptotic expansion of the function e^x using the sequence of functions $\{1, \sin x, \sin^2 x, \sin^3 x \cdots\}$ as $x \to 0$.

JU 13/10/10.