QUESTION 1. Consider the parabolic cylinder function, defined by

$$D_{-2m}(x) = \frac{1}{(m-1)!} x e^{-x^2/4} \int_0^\infty e^{-s} s^{m-1} (x^2+2s)^{-m-1/2} ds, \tag{1}$$

where m is some positive integer. Show that for real x and fixed m,

$$D_{-2m}(x) \sim x^{-2m} e^{-x^2/4}, \tag{2}$$

as $x \to \infty$.

QUESTION 2. Show that as $x \to \infty$

$$I(x) = \int_0^1 \frac{e^{-xt^n}}{1+t} dt \sim \frac{\Gamma(1/n)}{n x^{\frac{1}{n}}} \tag{3}$$

JU 29/10/11