Statement of Research

1 Academic background and motivation

As a Faculty of the Applied Mathematics Group at the University of Birmingham I am active in research in fluid mechanics. I am interested in developing mathematical/computational models to capture the essential physics behind problems in industry and engineering which revolve around fluid mechanics. I work closely with various interdisciplinary groups with the aim of applying my mathematical expertise. My current research projects include work on particle collisions, free surface hydrodynamics, rupture of liquid jets and films, oscillations in fuel cells and liquid curtain coating. I am currently the main supervisor to 4 PhD students and a co- supervisor of a further 2 PhD students in various different fields of fluid dynamics including free surface flow, fuel cell modelling, nonlinear oscillations and thin film flow. My PhD thesis was an investigation of the instability of curved liquid jets with applications to industrial prilling under the supervision of Prof. Stephen Decent (Mathematics) and Prof. Mark Simmons (Chemical Engineering). I was awarded a three year Fellowship in Hydrodynamics after my doctoral studies working with Prof. David Needham on the asymptotic structure of thin jets caused by the motion of solid boundaries near a free surface. During my fellowship I was also a sponsored visiting researcher at the High Speed Fluids Lab at the King Abdullah University of Science and Technology (KAUST) working alongside Dr. Jeremy Marston and Prof. Sigurdur Thoroddsen.

I have extensive experience of modelling free surface flows both via rigorous and often elaborate analytical mathematical techniques and computational methods. I am particularly interested in problems which prove inherently sophisticated, and therefore require detailed and rigorous mathematical treatment, due to for example singularities or other topological changes in flow structure and this together with a desire to tackle problems of relevance to industrialists has resulted in a number of high profile publications and numerous national and international seminar invitations.

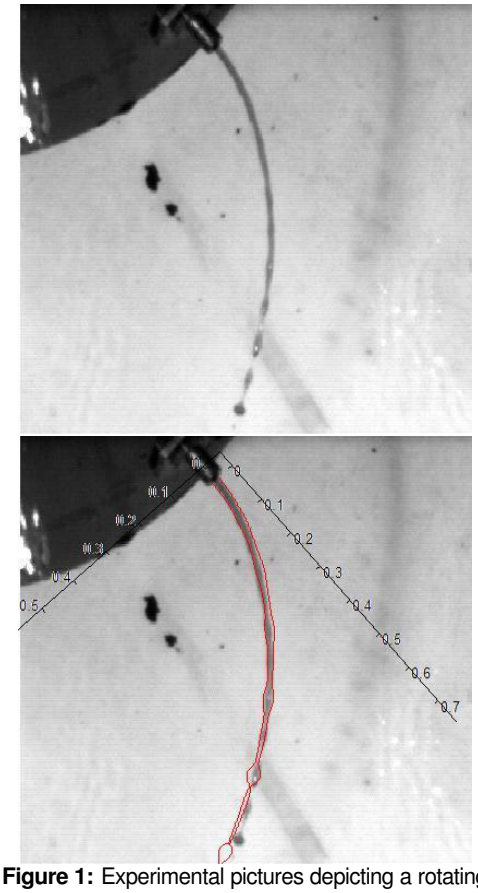

Figure 1: Experimental pictures depicting a rotating liquid jet emerging from an orifice located on a cylindrical drum. The curved jet breaks up and forms drops due to centrifugal instabilities. These drops are cooled and solidified to from pellets. A theoretical model (red lines in figure) developed in [1]-[6] is used to accurately predict location of breakup and drop sizes.

2 Research on Free Surface Flows

My primary work to date involves an examination of the flow structure and dynamics of a liquid thread towards breakup and droplet formation [1]. Most of this work has been motivated by industrial prilling where the need to produce fertilizer pellets from a rotating molten liquid jet is sought after and by applications in pharmaceutics where the breakup of compound liquid jets may be utilised to form compound capsules. The growth of waves along curved liquid jets (see Fig. 1 for a typical example of such jets) has been investigated through examination of a temporal instability analysis [2, 3, 4] and also a fully nonlinear/numerical analysis to explore breakup dynamics and droplet formation [5]. The roles of surfactants, gravity and non-Newtonian rheology have been considered as industrial extensions of these works (see [5, 6, 7, 8]). A computational algorithm based on the Lax-Wendroff method and developed by myself and has been used in [9, 10, 11] to investigate the local structure of rupture and compound or two-fluid droplet formation. The intricate geometry used in such works as well methods to investigate temporal stability has direct relevance to a whole host of related problems in thin film flows. The addition of surfactants and other body forces for a similar set-up has been investigated in [12] and [13].

More recently I have examined the flow structure in the locality of a free surface (having weak surface tension) and a moving solid boundary which is a classical problem in hydrodynamics (see [14]) and one which has relevance to water entry problems and droplet impact.

Curtain coating has been commonly used in a wide variety of engineering applications but the need to investigate this process for multilayer coating is increasingly more relevant as products become more and more sophisticated. In [18] a thorough experimental analysis of different regimes encountered in multilayer coating with up to three layers is presented. Additionally the growth of holes which rupture the curtain is analysed in the context of the Taylor-Culick opening speed.

3 Cavitation and squeeze flow in non-Newtonian fluids

In [15] we have conducted experiments to examine the flow-field in the region directly beneath a sphere, which impacts onto a thin, viscous film of oil, creating a squeeze flow. Using high-magnification and narrow depth-of-field, we have performed pseudo-micro-PIV at frame rates up to 30 kfps, which

provides direct measurements of the velocity field and enabled calculation of the principal deformation ratethe horizontal shear. Traditionally in previous studies, silicone oil (which is the fluid used in [15]) has been taken as Newtonian however in [15] we show through a theoretical and experimental approach that variations in viscosity due to shear are significant and as such need to be taken into account to provide accurate information.

The experimental results clearly show that there are high-shear regions close to the axis of symmetry, which, owing to the non-Newtonian nature of the oils, inevitably leads to significant viscosity reduction, up to 60% in some cases. This viscosity reduction is shown to lead to increased sphere penetration, incorporating the instantaneous sphere velocity and apparent viscosity, which can reduce by up to a half for some realisations. As such, it appears the impact Stokes number, taking a single-valued viscosity, is the correct non-dimensional number to characterise the impact and penetration for Newtonian fluids, but for shear-thinning fluids, a reduced viscosity value may be taken to estimate the deeper penetration observed.

Theoretical predictions based on a lubrication model for the Carreau fluid for the velocity plots (see Fig. 2) and for sphere penetration and minimum depths agree well with data obtained through experiments.

In [16] the onset of cavitation during the collision of a sphere with a solid surface covered with a layer of Newtonian liquid is investigated. The conventional theory dictates cavitation to initiate during depressurization, i.e. when the sphere rebounds from the solid surface. A synchronized dual-view high-speed imaging technique was used to show conclusive experimental evidence that confirms this scenario - namely - that cavitation occurs only after the sphere makes initial contact with the solid surface. This is further supported through the use of a numerical model based on lubrication theory to predict the occurrence of pressures below the vapour pressure. Similar to previous experimental observations for spheres released above the liquid surface, bubbles are formed on the sphere surface during entry into the liquid layer. These were found to squeeze radially outwards with the liquid flow as the sphere approached the solid surface, producing an annular bubble structure unrelated to cavitation. In contrast, spheres released below the liquid surface did not exhibit these patterns.

4 Nonlinear oscillations in fuel cells

The phenomenon of temporal, self-sustained oscillations which have been observed under quite general conditions in solid oxide fuel cells (SOFCs) has been investigated in [17]. The main idea behind this work is to uncover the fundamental mechanisms giving rise to the observed oscillations. To this end a model is developed based on the fundamental chemical kinetics and transfer processes which take place within the fuel cell. This leads to a three-dimensional dynamical system, which, under typical operating conditions, is rationally reduceable to a planar dynamical system. The structural dynamics of the planar dynamical system are studied in detail. Self-sustained oscillations are shown to arise through Hopf bifurcations in this planar dynamical system, and the key parameter ranges for the occurrence of such oscillations are identified.

In this work only the simplest hydrogen forming reaction is considered and we model the operation of an SOFC by the two half cell reactions which occur at opposite electrodes, separated by a solid electrolyte which is usually made from yttria-stabilised Zirconia. The reactions are, respectively,

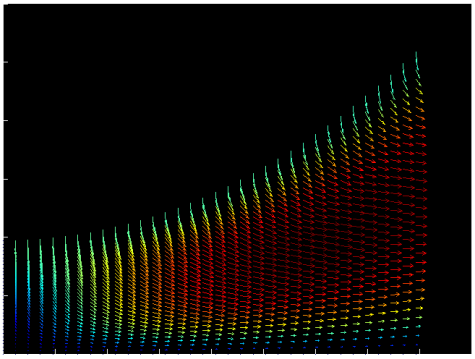

Figure 2: A numerically generated vector flow field of a viscous film in a thin region between a solid sphere and a solid wall. The fluid is assumed to have a shear dependent viscosity which is assumed to obey a Carreau type model. Strong shear is observed which has significant consequences on the viscosity of the fluid and consequently the motion of the sphere towards the wall.

$$H_2 + O^{2-} - 2e^- \rightarrow 2H_2O \qquad (anode)$$
$$O + 2e^- \rightarrow O^2 (cathode)$$

The electrolyte ideally acts as a barrier to anything other than the ions produced at the cathode via the half cell reaction above at temperatures approximately between $600 - 1000C$.

5 Taylor dispersion in tubes with Paraytec & Malvern Instruments

A key problem facing developers of formulations of biotherapeutic proteins is that of characterising key attributes using minimum volume of these high value materials. Proteins are formulated at high concentration and companies wish to find out about protein-protein interactions and properties of the solution, whilst constrained to the use of extremely small volumes (10 microlitres or less) of material. Usually, the proteins are injected into and driven by pressure through a capillary, and their concentration profiles visualised at two windows. Analysis of these profiles yields some information about protein size and solution viscosity.

In this project we established a theoretical framework for the modelling of concentration-dependent effects under the flow conditions relevant to Paraytecs TDA200 instrument (see [19]). In particular we adapted Taylor dispersion theory to allow diffusion to have a linear dependence on concentration and investigated potential approaches to the inclusion of concentration dependent viscosity. In our approach, we used the frontal injection mode where the sample is continually driven by pressure through the capillary and displaces the carrier solution. Numerical simulations using MATLAB (se Fig. 3) were used to develop an algorithm for determining the protein sizes from dispersion data using the Stokes-Einstein relation.

6 Rotating Disk Problem with Schlumberger

In 2011 as part of the First Industrial Study Group at KAUST I participated in a project put forward by Dr. Frank Chang from Schlumberger (see [20]). This problem was based on a

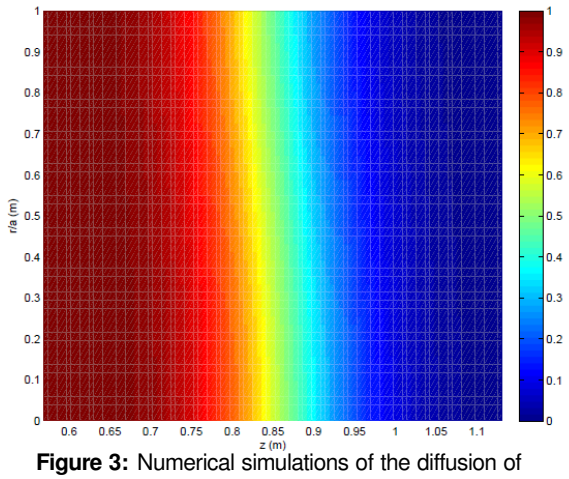

Figure 3: Numerical simulations of the diffusion of a sample (having initial distribution in the form of a step profile) moving in a cylindrical tube. Results from Taylor dispersion theory are used to investigate the modified case where viscosity is no longer constant but is concentration dependent.

Figure 4: Dimpled structures observed on carbonate rotating disk heads caused by mass transfer of acidic solutions during drilling process. Picture courtesy of Frank Chang (Schlumberger).

rotating disk apparatus which is widely used in the petroleum industry to study the kinetics of heterogeneous reactions such as interfacial calcium carbonate dissolution by acid. In practice, a carbonate rock disk is attached to a rotating shaft, and submerged in a solution of reactant (typically strong acid). The reactant is transferred to the rock surface by both convection and molecular diffusion with the overall reaction rate governed by the slower of these two processes.

Schlumberger were interested in understanding the formation of dimpled structures on their rotating drill heads (see Fig. 4) and in so doing they were interested in determining the best location for sampling of calcium concentrations in their experimental set-ups. Through a boundary layer analysis we were able to demonstrate the appearance of spiral pattern waves on the rotating head and from a theoretical/computational approach determine improved locations of sampling (see [20]).

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